ing. This generally results in smaller values of both the propagation constant and the surface transfer inductance.

References

[1] J. R. Wait, "Electromagnetic theory of the loosely braided coaxial cable, Part I," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-24, pp. 547–554, Sept. 1976.

[2] J. R. Wait and D. A. Hill, "Propagation along a braided coaxial cable in a circular tunnel," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, pp. 401–405, May 1975.

[3] J. Fontaine, B. DeMoulin, P. Degauque, and K. Gabillard, "Feasibility of radio communication in mine galleries by means of a coaxial cable having a high coupling impedance," in *1973 Proc. Thru-the-Earth Electromagnetics Workshop (Colorado School of Mines),* U. S. Bureau of Mines Contract G-133023, Final Rep., Dec. 31, 1973, pp. 130–139 (available from NTIS, Springfield, VA).

[4] P. Delogne and M. Safak, "Electromagnetic theory of the leaky coaxial cable," *Radio Electron. Eng.*, vol. 45, pp. 233–240, 1975.

[5] M. Abramowitz and I. A. Stegun, Eds., *Handbook of Mathematical Functions./* Washington, DC: National Bureau of Standards, AMS-55, 1964, p. 378.

[6] D. A. Hill and J. R. Wait, "Propagation along a coaxial cable with a helical shield," *IEEE Trans. Microwave Theory Tech.*, to be published.

[7] R. J. Slaughter, "Field leakage and crosstalk, with special references to radiating cables with perforated tape screens," *Radio Electron. Eng.*, vol. 45, pp. 248–252, 1975.

[8] R. W. Hamming, *Numerical Methods for Scientists and Engineers.* New York: McGraw-Hill, 1973, pp. 62–63.

[9] K. F. Casey, "Effects of braid resistance and weatherproofing jackets on coaxial cable shielding," Kansas State Univ., Manhattan, Report on Contract AFWL-TR-208 on DNA Subtask EB088; also, Interaction Note 192.

[10] ——, "On the effective transfer impedance of thin coaxial cable shields," *IEEE Trans. Electromagn. Compat.*, vol. EMC-18, pp. 110–116, May 1976.

[11] J. R. Wait and D. A. Hill, "Influence of spatial dispersion of the shield transfer impedance of a braided coaxial cable," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-25, pp. 72–74, Jan. 1977.

Improved Single and Multiaperature Waveguide Coupling Theory, Including Explanation of Mutual Interactions

RALPH LEVY, FELLOW, IEEE

***Abstract*—Bethe's small aperture coupling theory, modified by Cohn for large coupling apertures, is improved by including correction terms obtained by averaging the fields over the large aperture. Additionally, inclusion of nonempirical thickness correction factors derived previously by McDonald give coupling formulas which result in theoretical predictions for multiaperture couplers substantially in exact agreement with experiment (correcting small discrepancies previously noted by the author in a 1968 paper). This agreement is now so close that it becomes possible both to identify and explain the mutual interaction effects between closely spaced apertures in multiaperture couplers. It is shown that the mutual interaction is due to contradirectional (or backward) waves in the secondary arm, so that multiaperture interactions are manifested as elimination of the self-interactions of the individual apertures (since the high directivity of typical multiaperture couplers implies negligible backward wave amplitude).**

I. Introduction

THE THEORY OF microwave coupling by large apertures has developed in a number of stages, originating in Bethe's small aperture coupling theory of 1943 [1], [2]. A major extension of Bethe's work has been described by Cohn in 1952 [3], and enabled the theory to be applied to large apertures of finite thickness. Cohn recognized that a coupling aperture between two waveguides has an equivalent circuit representation involving lossless impedances, which must therefore obey Foster's reactance theorem. Hence to take account of the aperture resonance, the impedance was modified simply by inclusion of a factor $(1-f^2/f_0^2)$, where f is frequency and f_0 the resonant frequency of the aperture. The effect of finite thickness was taken into account by treating the aperture as a finite length of waveguide beyond cutoff. However it was noted that this thickness correction was somewhat empirical, and "effective thickness" factors had to be included to give reasonable agreement between theory and experiment.

The Bethe–Cohn theory was applied to the analysis and synthesis of multiaperture waveguide directional couplers by the author in 1968 [4]. It was shown to give excellent results for predicting the directivity of multiaperture couplers, and the coupling could be predicted to within 0.3 dB over most of a complete waveguide band. On the other hand, at high frequencies, between f/f_c values of 1.6 and 1.8, the discrepancy in coupling increased gradually from (typically) 0.3 dB to 0.7 dB, independently of the absolute

Manuscript received June 1, 1979, revised November 2, 1979. This paper was presented at the 1979 IEEE MTT-S Symposium, May 2, 1979, Orlando, FL.

The author is with Microwave Development Laboratories, Inc., Natick, MA 01760.

0018-9480/80/0400-0331$00.75

coupling value or the number of coupling apertures. A second unsatisfactory feature was the inclusion of the empirical correction factor for finite coupling wall thickness, a factor which was obtained by matching the theory of the experimental results to a considerable extent. Hence one could justifiably question the validity of the original theoretical basis, even though the design technique was superior to previous methods.

Fortunately, the thickness correction factor has now been obtained rigorously in a paper by McDonald [5] and it will be shown that inclusion of his thickness correction factors in a modified Bethe–Cohn aperture coupling theory gives precise results, requiring no empirical adjustments.

Meanwhile, a number of papers by Pandharipande and Das have been published, e.g., [6] which describes the coupling of a long rectangular transverse slot in the common broad wall between two rectangular waveguides. This work has the merit that there is no recourse to measured polarizabilities for slot apertures, whereas application of Bethe's method to slot coupling requires use of Cohn's measured polarizability data [7]. On the other hand, it should be noted that exact polarizability formulas are known for circular and elliptically shaped apertures [2]. A noteworthy result from [6] is the appearance of the Cohn apertures resonance factor $(1-f^2/f_0^2)$ as a natural consequence of the theory. Unfortunately it appears that this work may be valid only for slots near resonance [8], [9]. The evidence is conflicting, but the experimental results shown in [9, fig. 1] are clearly in error for small slot lengths. For example a centered transverse slot of length 5 mm, width 1 mm, and wall thickness 1.58 mm in WR90 has a coupling of about 55 dB rather than the 40 dB indicated in [9, fig. 1]. A second disadvantage is the apparent lack of generality of this theory, since each distinct aperture shape or field excitation involves solution of a fresh complex mathematical problem. The method does not build upon previous work, and thus no results are available for circular coupling apertures, where exact aperture polarizabilities are given by Bethe [2].

The approach adopted in this paper is to modify the Bethe–Cohn theory in order to correct the frequency dependence of the coupling. In this way it is possible to retain the work of the past few decades, thus avoiding adoption of radically different and more complex methods. An unexpected benefit resulting from the modification is a solution to the problem of mutual interaction between coupling apertures. This leads to a further conclusion that any future competitive theory must also be capable of taking this interaction into account.

II. Modified Bethe–Cohn Aperture Coupling Theory

A basic premise of the original Bethe theory was the assumption of an aperture small compared to a wavelength, so that the field could be considered as uniform over the aperture. The introduction of the aperture resonance term by Cohn results in a great improvement by guaranteeing correct results in the limiting cases of small apertures and resonant apertures. There is a significant improvement for intermediate cases also, but there seems little reason to assume that the theory will be precise for such cases.

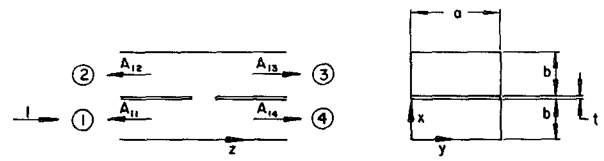

Fig. 1. Illustration of Bethe's small-aperture coupling theory for rectangular waveguides.

Now Bethe's theory assumes that the coupling is expressed in terms of electric and magnetic dipoles excited by fields which would be present at the aperture if it were not there. A natural extension is surely to *average* the field over the cross section of the aperture, resulting in a further correction term in addition to that derived by Cohn. This averaging process will not affect the limiting cases of infinitesimally small and resonant apertures. The derivation of the aperture polarizabilities involves integration of fields over the surface of the aperture, e.g., [10, p. 289, eq. (56)], but previous authors assumed the aperture to be small and the field was taken as a constant in evaluating the integral. Hence averaging the field over the aperture surface would be expected to give a closer approximation to the true situation.

The simplest way to describe the field averaging technique is by means of examples. Initially Bethe's equations will be reviewed for the single aperture shown in Fig. 1. A single aperture may consist of one or more holes or slots on a transverse plane, i.e., centered at $z=0$. Assuming rectangular waveguides propagating the dominant H_{10} mode, the expressions for the forward-coupled wave A_{13} and the contradirectional or backward-coupled wave A_{12} are

$$\left.\begin{matrix} A_{13} \\ A_{12} \end{matrix}\right\} = j\frac{2\pi}{ab\lambda_g}\left[\pm M_x H_x^{(1)} H_x^{(2)} + M_z H_z^{(1)} H_z^{(2)} - P E_y^{(1)} E_y^{(2)}\right] \tag{1}$$

where the plus sign is taken for A_{13} and the minus sign for A_{12}. M_x and M_z are the components of the magnetic polarizability of the aperture in the x and z directions, and P is the electric polarizability; $H_x^{(1)}$ is the magnitude of the x component of the magnetic vector in the first guide evaluated at the center of the aperture, while $H_x^{(2)}$ is the corresponding quantity defined for the second guide, assuming that the unit amplitude wave was incident in it. The definitions for the other field quantities in (1) are similar. When the guides are identical, $H_x^{(1)} = H_x^{(2)} = H_x$, etc., and the values of the field components are given by

$$H_x = -\sin\frac{\pi x}{a}\cdot e^{-j\gamma z}$$
$$H_z = j\frac{\lambda_g}{2a}\cos\frac{\pi x}{a}\cdot e^{-j\gamma z}$$
$$E_y = \frac{\lambda_g}{\lambda}\sin\frac{\pi x}{a}\cdot e^{-j\gamma z} \tag{2}$$

where the wave is traveling in the positive z direction. Substitution of these expressions in (1) gives the following equation for the forward- and backward-coupled waves at the aperture, i.e., at $z=0$:

$$\left.\begin{matrix}A_{13}\\A_{12}\end{matrix}\right\} = j\frac{2\pi}{ab\lambda_g}\left[\pm M_x\sin^2\frac{\pi x}{a} + M_z\left(\frac{\lambda_g}{2a}\right)^2\cos^2\frac{\pi x}{a} - P_y\left(\frac{\lambda_g}{\lambda}\right)^2\sin^2\frac{\pi x}{a}\right]. \tag{3}$$

For small apertures the polarizabilities M_x, M_z, and P_y are frequency-independent quantities having the dimensions of volume. In the original paper [4] it is shown that the various terms in (3) correspond to discrete equivalent circuit elements in series or shunt with two waveguides. An equivalent circuit is formed which enables analysis of multiaperture couplers to be carried out. This results also in expressions for the waves A_{11} and A_{14} of Fig. 1 to be derived. Large aperture effects are taken into account using a correction factor [4, eq. (12)] by which the aperture polarizabilities are multiplied, given by

$$K(f_0,A) = \frac{\tan(\pi f/2f_0)}{\pi f/2f_0}\cdot\exp\left[\frac{-2\pi tA}{\lambda_c}\sqrt{1-\frac{f^2}{f_0^2}}\right]. \tag{4}$$

The first term in (4), introduced by Cohn [3], takes account of the effects of aperture resonant frequency f_0, and is preferred to the alternate form $1/(1-f^2/f_0^2)$ since it seems more appropriate for a distributed circuit. The exponential term corrects for finite thickness t of the coupling wall, where A is an empirical factor which takes into account an apparent extra electrical thickness of the coupling wall. In the case of circular holes it was determined that A is a function of the hole diameter D, i.e.,

$$At = \alpha D + t \tag{5}$$

where α is a constant ~0.065 [4, eq. (15)]. This expression has now been justified by a solution of the problem of a small aperture of finite thickness by McDonald [5], an important advance which will be discussed in the Appendix. Henceforth it will be assumed that McDonald's expression for A replaces that given by (5). In the case of long slots a similar McDonald expression is used.

For the first example of the field-averaging technique, consider an aperture consisting of transverse and longitudinal slots in the common wall of two identical coupled waveguides, as shown in Fig. 2. Dealing first with the centered transverse slot, the coupling is mainly via the H_x field if the slot width w is small. Referring to (1) it is seen

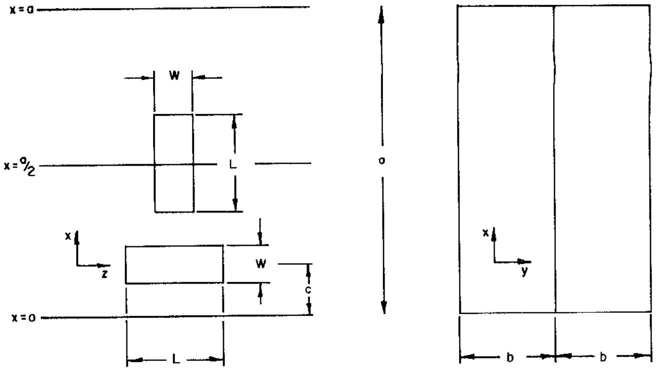

Fig. 2. Longitudinal and transverse slots coupling common broad wall of two waveguides (slots need not be identical).

that the coupling depends on a field $H_x^{(1)}\cdot H_x^{(2)} = H_x^2$, with H_x given by (2). This is now to be averaged over the slot to give an additional *field averaging correction factor*

$$A(H_x^2) = \int_{(a-L)/2}^{(a+L)/2}\sin^2\frac{\pi x}{a}\,dx\Big/\int_{(a-L)/2}^{(a+L)/2}dx$$
$$= \frac{1}{2}\left(1+\frac{\sin(\pi L/a)}{\pi L/a}\right). \tag{6}$$

The case of the narrow longitudinal slot is somewhat different, since the main coupling is via the H_z field component. Conventionally the exponential term is ignored, but in the averaging procedure it is necessary to consider the instantaneous sinusoidal variation of the field over the aperture in the direction of propagation z. Hence the z variation must be taken as $\cos(2\pi z/\lambda_g)$, with $z=0$ at the center of the slot. Finally the averaging factor for H_z^2 is given by

$$A(H_z^2) = \int_{-L/2}^{L/2}\cos^2\frac{2\pi z}{\lambda_g}\,dz\Big/\int_{-L/2}^{L/2}dz$$
$$= \frac{1}{2}\left(1+\frac{\sin(2\pi L/\lambda_g)}{2\pi L/\lambda_g}\right). \tag{7}$$

A significant difference between $A(H_x^2)$ and $A(H_z^2)$ now appears, since $A(H_x^2)$ represents a small frequency-*independent* attenuation, whereas $A(H_z^2)$ represents frequency-*dependent* attenuation.

The standard correction factor (4) is now modified simply by multiplication by the appropriate additional correction factor (6) or (7). These are always <1, so that weaker coupling is predicted compared with the unmodified Bethe–Cohn theory. In the case of apertures having finite dimensions in the z direction, an additional frequency dependence occurs. This results in looser coupling at higher frequencies, as required experimentally.

In cases where the apertures have more complicated shapes than the simple rectangles of Fig. 2, e.g., round-ended slots, it may be necessary to carry out the integration numerically, but this is easily incorporated in a computer program. Since practical coupling slots are often quite wide it is also desirable to take all three field components into account, and if incorporated within the

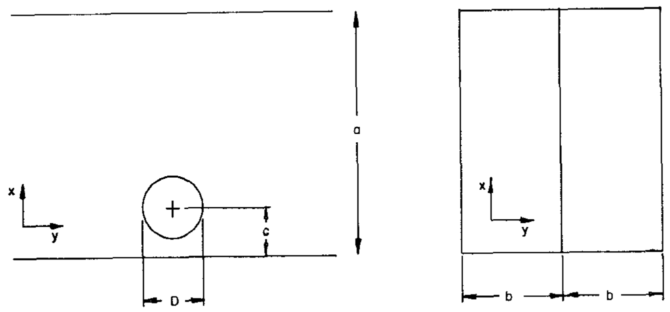

Fig. 3. Circular hole coupling aperture.

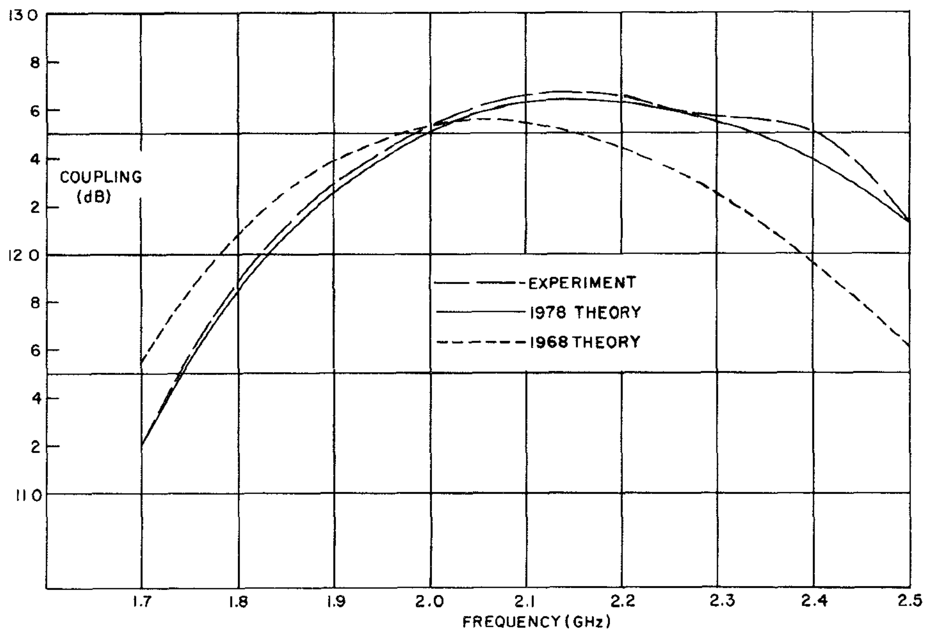

Fig. 4. WR430 six-aperture coupler (cf. [4, fig. 12]).

computer program this scarcely complicates practical implementations.

The more general averaging is illustrated by the popular circular coupling aperture shown in Fig. 3. Here the averaging factor for the H_x field is given by

$$A(H_x^2)=\int\int \sin^2\frac{\pi x}{a}\cos^2\frac{2\pi z}{\lambda_g}\,dx\,dz \Big/ \int\int \sin^2\frac{\pi c}{a}\,dx\,dz$$

$$=\int\int \sin^2\frac{\pi x}{a}\cos^2\frac{2\pi z}{\lambda_g}\,dx\,dz \Big/ \left(\frac{\pi}{4}D^2\sin^2\frac{\pi c}{a}\right) \quad (8)$$

where the double integrations are performed over the surface of the aperture. Similar expressions may be stated for the H_z and E_y field components.

III. Practical Results

The new field-averaging correction factors combined with McDonald's thickness correction factor give theoretical results which are in agreement with experiment within experimental error. This statement has been proven for a majority of the cases examined in all waveguide sizes, for coupling values ranging from 3 dB through 50 dB, for apertures consisting of circular holes or round-ended slots, and for any number of apertures in the range 3–30. (The results for one or two apertures are exceptional, and lead to significant conclusions described later.)

A typical example is the six-aperture WR-430 12.5 dB broad-wall coupler given previously in 1968, [4, fig. 12]. This is reproduced in Fig. 4 with the addition of the latest theory (derived in 1978), and shows agreement for coupling to within 0.1 dB across the entire waveguide band. The diameters of the apertures in this symmetrical coupler are 1.08, 1.419, and 1.589 in from end to center, with a spacing of 1.917 in. It is seen that the apertures are of appreciable size relative to the quarter-wavelength spacing, yet the directivity is greater than 43 dB over the band [4, fig. 12].

It will be apparent that results for absolute coupling will not always be as precisely defined as ± 0.1 dB in practice, particularly for couplers made in small waveguide sizes. Errors in coupling as large as 0.4 dB may be encountered, but it should be realized that this can be produced by a coupling wall thickness change of only 0.003 in. Coupling changes of this order are frequently noted in measurements made before and after brazing, indicating the practical tolerance problems encountered. A tolerance of

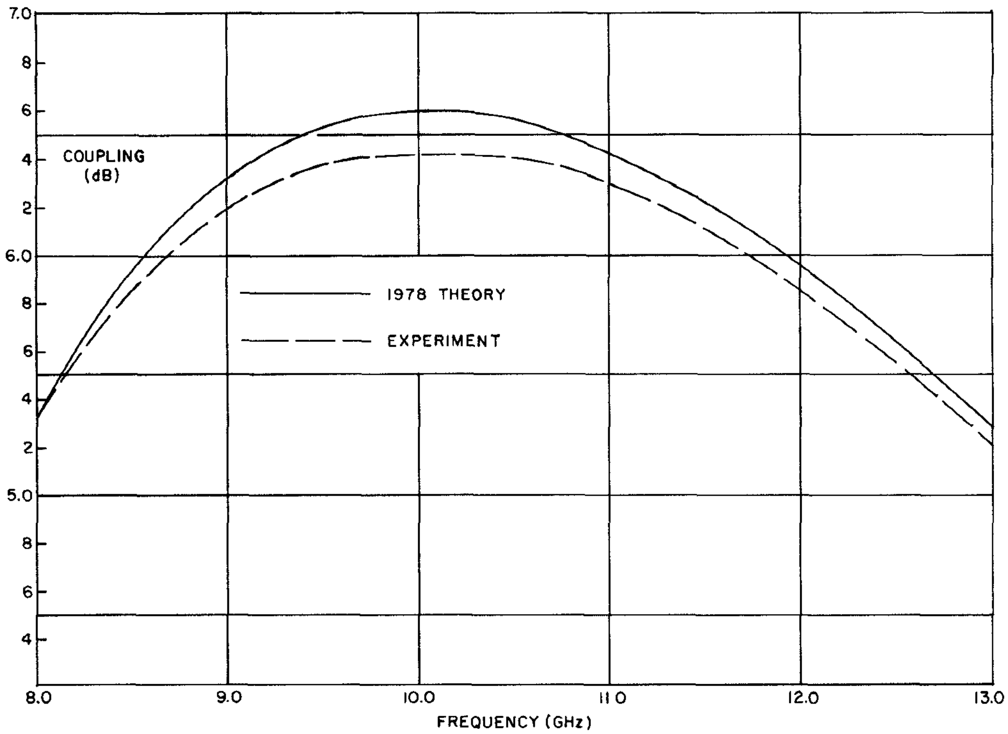

Fig. 5. WR90 15-aperture coupler using transverse and longitudinal slots in the common broad wall.

± 0.001 in on slot or hole dimensions of the order of 0.4 in may cause an error of approximately ± 0.1 dB in coupling, provided that the tolerance errors are all in the same direction. Discrepancies may arise also due to either burrs or rounded edges on the apertures, these being equivalent to an effective change in common wall thickness. Fortunately the very important minimum-coupling frequency appears to be well predicted in all cases, so that the coupling "balance" is unaffected by tolerances.

A second example illustrating the use of transverse and longitudinal slots [7] is shown in Fig. 5, and here the agreement between theory and experiment is better than 0.2 dB in absolute terms, or ± 0.1 dB if the absolute coupling value is adjusted by 0.1 dB.

These results have been obtained using a theory which contains *no* empirical factors. McDonald's theory for thick apertures combined with the field-averaging factors eliminates the previous rather unsatisfactory empirical corrections. Although McDonald's thick aperture theory [5] had been available for several years, it was difficult to apply while the theoretical coupling curves were displaced in frequency from the experimentally determined vales. It requires the field averaging correction to prove its validity, and vice versa.

IV. The Mutual Interaction Problem in Multiaperture Couplers

It is now interesting to apply the latest theory to directional couplers consisting of a single aperture. In the 1968 paper this was carried out for transverse double-hole apertures [4, fig. 6] of four different sizes (the double-hole is equivalent to a single aperture in the sense of having an equivalent circuit lumped at one point, i.e., at $z=0$). The agreement between theory and experiment then was no worse than ± 0.35 dB, but it must be recalled that the theory was made to fit the experiment as closely as possible by adjusting the empirical function for the effective thickness factor (5). When the 1968 theory was applied to multiaperture couplers the deviation between theory and experiment became worse (e.g. Fig. 4 of this paper, 1968 theory). We have seen that for multiaperture couplers this error is substantially reduced by application of field averaging.

A comparison between theory and experiment for the same set of single aperture couplers presented in 1968 is now shown in Fig. 6 for the new (1978) theory. The experimental data was repeated to confirm the earlier measurements, and no significant differences were observed. It is surprising to observe the increased deviation between theory and experiment as the apertures increase in size, the deviation being ± 0.6 dB for the 0.282-in diameter holes, compared with ± 0.35 dB for the 1968 results. Yet the 1978 results predict multiaperture couplers containing similar size holes almost perfectly, whereas the 1968 results are comparatively poor for multiaperture couplers. This paradox is further illustrated in the results of Fig. 7 which show the results of cascading sets of apertures, all having the 0.282-in diameter. The single aperture result is repeated, and compared with results for two and three apertures. As expected the agreement between theory and experiment improves as the number of apertures cascaded is increased, since it is known in

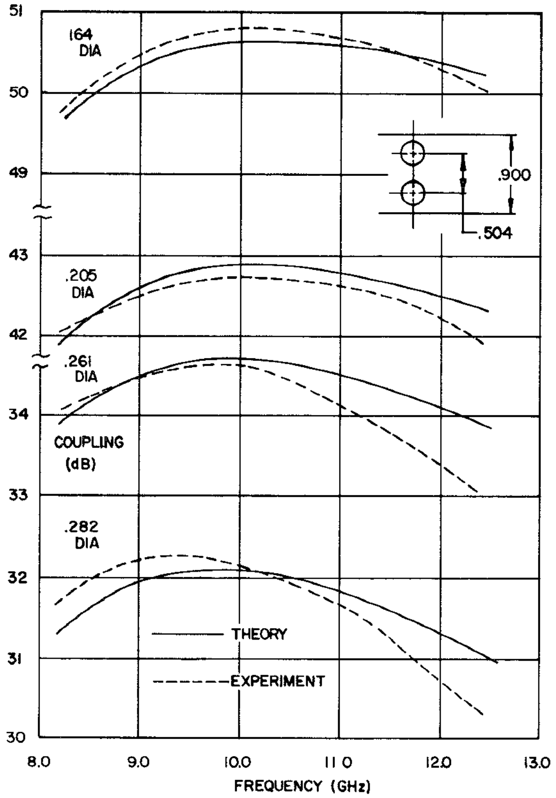

Fig. 6. Coupling of a single set of double-hole apertures.

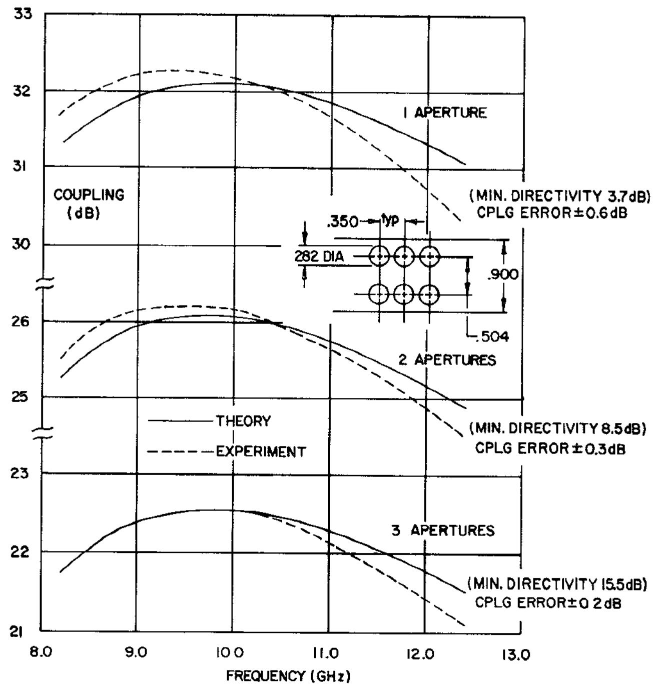

Fig. 7. Coupling of one, two, and three sets of double-hole apertures, hole diameter 0.282 in, transverse spacing 0.504 in.

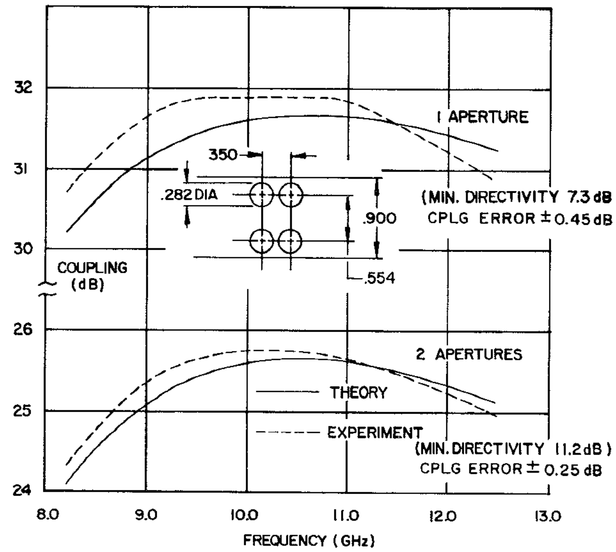

Fig. 8. Coupling of one and two sets of double-hole aperture, hole diameter 0.282 in, transverse spacing 0.554 in.

advance that multiaperture couplers are predicted almost perfectly.

The solution of this paradox becomes probable after one observes the relationship between the measured minimum directivity (occurring at the high frequency end of the band) and the coupling deviations. These are shown in Fig. 7, and indicate a substantial improvement in coupling accuracy as the directivity increases. The experiment was repeated with apertures having a different transverse spacing where the directivity is considerably better, as shown in Fig. 8. For these cases the directivity curve is approximately balanced, i.e., the directivity reaches its worst value at either end of the band. It is observed that the coupling deviation is improved for both the one- and two- aperture cases as compared with the corresponding cases of Fig. 7. The relationship between the minimum directivity and the coupling deviation forms a smooth curve, Fig. 9, where the results from both Figs. 7 and 8 have been plotted.

It seems clear that the new theory has taken the effect of aperture interactions into account, but requires high directivity to give good results. When the directivity is poor, there is a contradirectional or backward wave in the secondary arm which interferes with the assumed form of electromagnetic field in the aperture, namely that which would be present if the aperture were not there. When a few apertures are cascaded, the directivity is improved and the effect is to *regularize* the fields on either side of the aperture. This is simply the condition under which the field averaging process may be expected to give excellent results. It is remarkable that in this explanation of aperture interaction it is the *single* aperture which is to be regarded as inherently self-interacting, and the multiaperture "interaction" is explained as a *reduction to zero of the self-interaction.*

It is expected that this result should prove to be a challenge to other workers who prefer a more rigorous electromagnetic approach than that adopted here. Any subsequent theory must be capable of explaining the

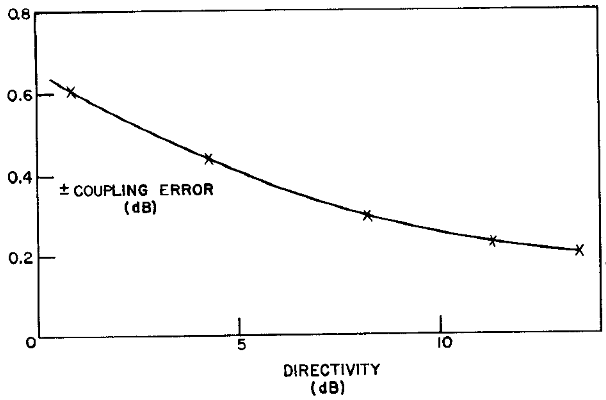

Fig. 9. Relationship between coupling deviations and minimum directivity for the couplers of Figs. 7 and 8.

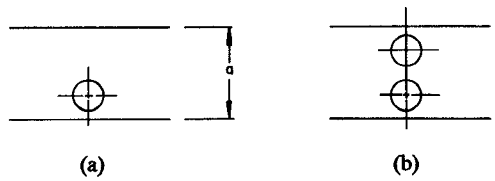

Fig. 10. (a) Broad-wall single-hole single-aperture coupler. (b) Double-hole single aperture coupler.

aperture interaction effects. Any theory based on the single aperture problem must be capable of predicting the interaction, possibly by identifying a term or sets of terms in the final expression as responsible for self-interaction coupling. Additionally, any new theory must take finite thickness effects properly into account as in [5], a feature which has been sadly lacking in most previous work.

In the 1968 paper it was conjectured that the effects described here might be caused by transverse interaction between holes in a double-hole aperture [4, p. 999]. Indeed the results for single aperture couplers consisting of a single coupling hole, as illustrated in Fig. 10(a), show smaller errors between theory and practice compared with the corresponding double-hole aperture, shown in Fig. 10(b), even though the directivity is about the same in the two cases. The hole diameters are all equal in this comparison. However the coupling for the single-hole single aperture is 6 dB less than that of the double-hole single aperture. Hence the field in the aperture may be expected to be perturbed less in the single-hole case, i.e., the effect is dependent on coupling strength as well as directivity. This assumption is reinforced by the observation that the results for multiaperture couplers consisting of a *single* row of holes are predicted as well as those for the double row cases. It is also worth noting that the coupler of Fig. 5 uses triple slot apertures, with two longitudinal slots for each centered transverse slot. These facts indicate that the transverse interaction effects are probably negligible, and the measurements are consistent with the self-interaction theory.

V. Conclusions

The Bethe–Cohn theory for large aperture waveguide couplers may be improved and refined by taking account of the field variations over the surface of the coupling aperture. In conjunction with McDonald's thick-aperture theory this results in a highly precise method for analysis (and hence synthesis) of multiaperture couplers, with coupling prediction typically within 0.2 dB of practice. The improved precision leads to identification of mutual interaction effects in multiaperture couplers, and to the conclusion that these are to be regarded as a reduction to zero of (quantitatively unidentified) self-interaction terms which are inherent to the individual apertures, rather than to conventional evanescent field interactions.

Further work in this general area of large aperture coupling would include effects of the side walls of the waveguide (probably rather small, and similar to transverse interaction), and coupling between waveguides and resonant cavities, etc. In the latter case there is no directivity concept as in the case of a four-port network, and different considerations may apply when evaluating the field-averaging technique. Identification of the self-interaction effect for a single aperture and prediction of the single-aperture coupling characteristics would be interesting.

VI. Appendix

McDonald Thick-Aperture Theory

The McDonald thick-aperture coupling theory is derived for very small apertures, and gives results which are stated as exact in the limit when the aperture dimensions tend to zero [5, p. 12]. In the case of a circular coupling hole of radius R and thickness t the electric and magnetic polarizabilities are given by

$$P = -C_E R^3$$
$$M = C_H R^3 \tag{9}$$

where

$$C_E = F_{CE} \cdot \tfrac{2}{3} \exp(-2.4048 t/R) \tag{10}$$
$$C_H = F_{CH} \cdot \tfrac{4}{3} \exp(-1.8421 t/R). \tag{11}$$

F_{CE}, F_{CH} are functions of the coupling wall thickness t, and are unity for $t=0$. As t increases F_{CE} becomes asymptotic to 0.825 and F_{CH} to 0.839. Hence these factors give an effective wall thickness which is larger than the actual wall thickness (assuming use of the simple exponential evanescent mode decay term), as expected. Equations (10) and (11) are seen to be frequency independent, because the aperture is assumed to be so small that the guide wavelength beyond cutoff in the aperture is equal to the aperture cutoff wavelength. In the case of large apertures it is necessary to modify McDonald's theory by using the correct frequency dependence, and

(9)–(11) become

$$P=\frac{2}{3}R^3\cdot\exp\left[\frac{-2\pi A_e t}{\lambda_{ce}}\sqrt{1-\frac{f^2}{f_{ce}^2}}\right] \quad (12)$$

$$M=\frac{4}{3}R^3\cdot\exp\left[\frac{-2\pi A_m t}{\lambda_{cm}}\sqrt{1-\frac{f^2}{f_{cm}^2}}\right] \quad (13)$$

where $f_{ce}, \lambda_{ce}, f_{cm}, \lambda_{cm}$ are the cutoff frequencies and wavelengths for the E_{10} and H_{11} modes, respectively, i.e.,

$$\lambda_{ce}=2.613R \quad (14)$$

$$\lambda_{cm}=3.412R. \quad (15)$$

A_e and A_m are derived from curves given by McDonald, e.g., [5, fig. 5], and these may be fitted to the following piecewise linear approximations:

$$C_E=\begin{cases}10\uparrow(-1.0551t/R-0.23657), & \text{for } t/R>0.2\\ 10\uparrow(-1.3992t/R-0.1675), & \text{for } t/R<0.2\end{cases} \quad (16)$$

$$C_H=\begin{cases}10\uparrow(-0.80485t/R+0.0594), & \text{for } t/R>0.2\\ 10\uparrow(-1.14148t/R+0.1268), & \text{for } t/R<0.2.\end{cases} \quad (17)$$

Now to derive A_e (for example), we form the equivalence

$$\exp\left(\frac{-2\pi A_e t}{2.613R}\right)=\frac{3}{2}C_E. \quad (18)$$

Hence for $t/R>0.2$, we have

$$\frac{-2\pi A_e t}{2.613R}=\ln\left[\frac{3}{2}\cdot 10\uparrow\left(\frac{-1.0551t}{R}-0.23657\right)\right] \quad (19)$$

leading to

$$A_e t=1.0103t+0.0579R, \qquad \text{for } t/R>0.2. \quad (20)$$

Similarly we have

$$\frac{-2\pi A_m t}{3.412R}=\ln\left[\frac{3}{4}\cdot 10\uparrow\left(\frac{-0.80485t}{R}+0.0594\right)\right], \qquad \text{for } \frac{t}{R}>0.2 \quad (21)$$

giving

$$A_m t=1.0064t+0.0819R, \qquad \text{for } t/R>0.2. \quad (22)$$

The expressions for $t/R<0.2$ are derived similarly. For most commonly used apertures the condition $t/R>0.2$ will be satisfied.

It must be noted carefully that (12) and (13) give the correction effect for finite thickness only. The Cohn large aperture correction term as in (4) and the field-averaging term as, for example, in (8) must also be included. All terms are simply multiplied together to give the overall effective aperture polarizability.

It is interesting to compare the thickness correction factors of (20) and (22) to the empirical relationship used in the 1968 paper, [4, eq. (15)] and (5). The field-averaging correction process gives an attenuation which has the effect of decreasing the value of α in the empirical relationship, and prior to the incorporation of McDonald's theory it was established that α should be reduced to approximately 0.03 to 0.04 in order to give the correct absolute coupling value. Expressed in terms of radius rather than diameter, we have the approximate relationship

$$At=t+0.07R. \quad (23)$$

This equation was derived *before* the derivation of the extended McDonald correction factors (20) and (22), and was tested on many practical cases. The theoretically justified McDonald factors give even closer agreement on average between theory and experiment. The justification of the earlier empirical expression is one of the most satisfying aspects of this development.

Similar effective thickness factors for rectangular slots may be derived from McDonald's results [5, p. 12].

Acknowledgment

The author is grateful to R. Samarco, E. Scollins, and H. Krauss for experimental results, to Dr. H. J. Riblet for supporting facilities, to Prof. A. A. Oliner for discussions held at the 1979 IEEE MTT-S International Symposium, where this paper was first presented, and to Dr. N. A. McDonald for a valuable correspondence.

References

[1] H. A. Bethe, "Theory of side windows in waveguides," Massachusetts Inst. of Tech., Cambridge, MA, M.I.T. Rad. Lab. Rep. No. 43-27, Apr. 1943.
[2] H. A. Bethe, "Theory of diffraction by small holes," *Phys. Rev.* vol. 66, pp. 163–182, Oct. 1944.
[3] S. B. Cohn, "Microwave coupling by large apertures," *Proc. IRE*, vol. 40, pp. 696–699, June 1952.
[4] R. Levy, "Analysis and synthesis of waveguide multi-aperture directional couplers," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, pp. 995–1006, Dec. 1968.
[5] N. A. McDonald, "Electric and magnetic coupling through small apertures in shield walls of any thickness," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 689–695, Oct. 1972.
[6] V. M. Pandharipande and B. N. Das, "Coupling of waveguides through large apertures," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-26, pp. 209–212, Mar. 1978.
[7] S. B. Cohn, "Determination of aperture parameters by electrolytic-tank measurements," *Proc. IEEE*, pp. 1416–1421, Nov. 1951.
[8] A. J. Sangster, "Slot coupling between uniform rectangular waveguides," *IEEE Trans. Microwave Theory and Tech.*, vol. MTT-27, pp. 705–707, July 1979.
[9] V. M. Pandharipande and B. N. Das, "Comments on coupling of waveguides through large aperture," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-27, pp. 707–708, July 1979.
[10] R. E. Collin, *Field Theory of Guided Waves.* New York: McGraw-Hill, 1960.
[11] H. J. Riblet and T. S. Saad, "A new type of waveguide directional coupler," *Proc. IRE*, vol. 36, pp. 61–64, Jan. 1948.
[12] N. A. McDonald, "Electromagnetic coupling through small apertures," Dep. of Elec. Eng., Univ. Toronto, Toronto, Ontario, Canada, Res. Rep. 45, 1971.